

# LABSMC: Monte Carlo event generator for large-angle Bhabha scattering

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## Abstract

A Monte Carlo event generator is presented. An original algorithm is developed to simulate electron–positron scattering at energies and momentum transferred much more than the electron mass. The first-order electroweak radiative corrections are included completely. Higher order corrections are taken into account by means of electron structure functions.

PACS: 12.20.–m Quantum electrodynamics, 12.20.Ds Specific calculations

## 1 Introduction

The process of electron–positron (Bhabha) scattering was studied both theoretically and experimentally since many years [1]. It has almost pure electrodynamical nature and therefore could be described with very high precision by means of perturbative QED. The process is commonly used at  $e^+e^-$  colliders for luminosity measurements, because it has a large cross section and can be measured very accurately. The modern experimental technique of luminosity measurements reaches the one per mille level of accuracy, or even better, as at LEP1 [2]. This is a challenge for the theory. At the Born and one-loop levels the process was investigated in detail (see papers [3, 4, 5] and references therein), taking into account both QED and electroweak effects. The radiative corrections in the first order in the fine structure constant  $\alpha$  become insufficient now, one has to take into account higher order effects. In order to meet the requirements of experiments one has also to implement the results of analytical calculations into a Monte Carlo event generator.

In this note we present an event generator, based on the approach [6], how to merge the complete  $\mathcal{O}(\alpha)$  result with the leading logarithmic corrections in higher orders. The structure of the paper is as follows. In the next section we give the master formula for description of large-angle Bhabha scattering and decompose it into 13 parts of different kinematics. The consequent structure of the Monte Carlo code is described in Sec. 3. The main options and parameters of the program are presented in Sec. 4. Numerical results and the precision achieved are discussed in the Conclusions.

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## 2 The master formula

The reaction

$$e^-(p_1) + e^+(p_2) \rightarrow e^-(p'_1) + e^+(p'_2) + (n\gamma) \quad (1)$$

will be considered in the centre-of-mass reference frame of the incoming particles.

Let us start with the master formula in the form as given in paper [7]:

$$\begin{aligned} \frac{d\sigma^{e^+e^- \rightarrow e^+e^-(\gamma)}}{d\Omega_-} &= \int_{\bar{z}_1}^1 dz_1 \int_{\bar{z}_2}^1 dz_2 \mathcal{D}(z_1) \mathcal{D}(z_2) \frac{d\tilde{\sigma}_0(z_1, z_2)}{d\Omega_-} \left(1 + \frac{\alpha}{\pi} K_{SV}\right) \Theta \\ &\times \int_{y_{\text{th}}}^{Y_1} \frac{dy_1}{Y_1} \int_{y_{\text{th}}}^{Y_2} \frac{dy_2}{Y_2} \mathcal{D}\left(\frac{y_1}{Y_1}\right) \mathcal{D}\left(\frac{y_2}{Y_2}\right) \\ &+ \frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx}{x} \left\{ \left[ \left(1 - x + \frac{x^2}{2}\right) \ln \frac{\theta_0^2(1-x)^2}{4} + \frac{x^2}{2} \right] 2 \frac{d\sigma_0^{\text{Born}}}{d\Omega_-} \right. \\ &+ \left[ \left(1 - x + \frac{x^2}{2}\right) \ln \frac{\theta_0^2}{4} + \frac{x^2}{2} \right] \left[ \frac{4\alpha^2}{s(1-x)^2[2-x(1-c)]^4} \right. \\ &\times \left. \left. \left( \frac{3-3x+x^2+2x(2-x)c+c^2(1-x+x^2)}{1-c} \right)^2 \right. \right. \\ &+ \left. \left. \frac{4\alpha^2}{s[2-x(1+c)]^4} \left( \frac{3-3x+x^2-2x(2-x)c+c^2(1-x+x^2)}{1-c} \right)^2 \right] \right\} \Theta \\ &- \frac{\alpha^2}{4s} \left( \frac{3+c^2}{1-c} \right)^2 \frac{8\alpha}{\pi} \ln(\cot \frac{\theta}{2}) \ln \frac{\Delta\varepsilon}{\varepsilon} + \frac{\alpha^3}{2\pi^2 s} \int_{\substack{k^0 > \Delta\varepsilon \\ \pi - \theta_0 > \theta > \theta_0}} \frac{WT}{4} \Theta \frac{d\Gamma}{d\Omega_-}, \end{aligned} \quad (2)$$

$$\begin{aligned} Y_1 &= \frac{2z_1 z_2}{z_1 + z_2 - c(z_1 - z_2)}, & Y_2 &= \frac{z_1^2 + z_2^2 - (z_1^2 - z_2^2)c}{z_1 + z_2 - c(z_1 - z_2)}, \\ \bar{z}_1 &= \frac{y_{\text{th}}(1+c)}{2 - y_{\text{th}}(1-c)}, & \bar{z}_2 &= \frac{z_1 y_{\text{th}}(1-c)}{2z_1 - y_{\text{th}}(1+c)}. \end{aligned}$$

Step functions  $\Theta$  represent any possible cuts on the phase space of the corresponding variables. By  $K_{SV}$  we denoted the so-called  $K$ -factor<sup>1</sup>, which comes from virtual and soft radiative corrections, and therefore it can be factorized at the Born cross section  $d\tilde{\sigma}_0$ ,

$$\begin{aligned} K_{SV} &= -1 - 2\text{Li}_2(\sin^2 \frac{\theta}{2}) + 2\text{Li}_2(\cos^2 \frac{\theta}{2}) + \frac{1}{(3+c^2)^2} \left[ \frac{\pi^2}{3} (2c^4 - 3c^3 - 15c) \right. \\ &+ 2(2c^4 - 3c^3 + 9c^2 + 3c + 21) \ln^2(\sin \frac{\theta}{2}) - 4(c^4 + c^2 - 2c) \ln^2(\cos \frac{\theta}{2}) \\ &- 4(c^3 + 4c^2 + 5c + 6) \ln^2(\tan \frac{\theta}{2}) + 2(c^3 - 3c^2 + 7c - 5) \ln(\cos \frac{\theta}{2}) \\ &+ \left. 2(3c^3 + 9c^2 + 5c + 31) \ln(\sin \frac{\theta}{2}) \right]. \end{aligned} \quad (3)$$

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<sup>1</sup>The last term in the square brackets of the expression for the  $K$ -factor in Ref. [7] is incorrect.

The *shifted (boosted)* Born cross section (with vacuum polarization effect taken into account) reads

$$\begin{aligned}
\frac{d\tilde{\sigma}(z_1, z_2)}{d\varphi dc} &= \frac{4\alpha^2}{sa^2} \left[ \frac{a^2 + z_2^2(1+c)^2}{(1-\Pi(\tilde{t}))^2 2z_1^2(1-c)^2} + \frac{z_1^2(1-c)^2 + z_2^2(1+c)^2}{|1-\Pi(\tilde{s})|^2 2a^2} \right. \\
&\quad \left. - \operatorname{Re} \frac{z_2^2(1+c)^2}{(1-\Pi(\tilde{t}))(1-\Pi(\tilde{s}))^* a z_1(1-c)} \right], \\
\tilde{t} &= -\frac{1}{2} s z_1 Y_1 (1-c), \quad \tilde{s} = s z_1 z_2, \quad Y_1 = \frac{2z_1 z_2}{a}, \\
a &= z_1 + z_2 - (z_1 - z_2)c.
\end{aligned} \tag{4}$$

For detailed notation and the derivation look in Ref. [7]. The current version of the code includes also the contributions of  $Z$ -exchange and  $Z$ - $\gamma$  interference as well as the relevant set of the first order weak radiative corrections to be described elsewhere.

We quoted the complete expression; now we are going to discuss it and decompose into a form, suitable for an event generator.

The first term of Eq. (2) is written in the form of the Drell–Yan cross section. The leading logarithmic corrections are accounted by means of the  $\mathcal{D}$ -functions, which are the kernel functions of the Dokshitzer–Gribov–Altarelli–Parisi–Lipatov evolution equations. The rest supplements the sub-leading terms, which come from the straightforward calculations in the  $\mathcal{O}(\alpha)$  order.

The non-singlet electron structure function is expanded in the series in  $\alpha$ :

$$\begin{aligned}
\mathcal{D}(z) &= \delta(1-z) + \frac{\alpha}{2\pi} (L-1) P^{(1)}(z) + \left( \frac{\alpha}{2\pi} \right)^2 \frac{(L-1)^2}{2!} P^{(2)}(z) + \dots, \\
P^{(1,2)}(z) &= \lim_{\Delta \rightarrow 0} \left\{ \delta(1-z) P_{\Delta}^{(1,2)} + \Theta(1-\Delta-z) P_{\Theta}^{(1,2)}(z) \right\}, \\
P_{\Delta}^{(1)} &= 2 \ln \Delta + \frac{3}{2}, \quad P_{\Theta}^{(1)}(z) = \frac{1+z^2}{1-z}, \quad P_{\Delta}^{(2)} = \left( 2 \ln \Delta + \frac{3}{2} \right)^2 - \frac{2\pi^2}{3}, \\
P_{\Theta}^{(2)}(z) &= 2 \left[ \frac{1+z^2}{1-z} \left( 2 \ln(1-z) - \ln z + \frac{3}{2} \right) + \frac{1+z}{2} \ln z - 1 + z \right], \\
\mathcal{D}(z) &= \delta(1-z) \mathcal{D}_{\Delta} + \Theta(1-\Delta-z) \mathcal{D}_{\Theta}(z), \\
\mathcal{D}_{\Delta} &= 1 + \mathcal{D}_{\Delta}^{[\alpha]} + \mathcal{D}_{\Delta}^{[\alpha^2]} + \dots, \quad \mathcal{D}_{\Theta}(z) = \mathcal{D}_{\Theta}^{[\alpha]}(z) + \mathcal{D}_{\Theta}^{[\alpha^2]} + \dots, \\
\mathcal{D}_{\Delta}^{[\alpha]} &= \frac{\alpha}{2\pi} (L-1) P_{\Delta}^{(1)}, \quad \mathcal{D}_{\Theta}^{[\alpha]}(z) = \frac{\alpha}{2\pi} (L-1) P_{\Theta}^{(1)}(z), \\
\mathcal{D}_{\Delta}^{[\alpha^2]} &= \left( \frac{\alpha}{2\pi} \right)^2 \frac{(L-1)^2}{2!} P_{\Delta}^{(2)}, \quad \mathcal{D}_{\Theta}^{[\alpha^2]}(z) = \left( \frac{\alpha}{2\pi} \right)^2 \frac{(L-1)^2}{2!} P_{\Theta}^{(2)}(z), \quad L = \ln \frac{s}{m_e^2}.
\end{aligned} \tag{5}$$

Here  $z$  means the energy fraction of an electron just before it emitted a collinear photon. The leading logarithmic corrections due to electron–positron pair production can be easily added within the same formalism [7, 8].

## 2.1 The kinematical regions

Our idea is to decompose the formula according to different types of the final state kinematics. Each part of the decomposition is a particular contribution to the total Bhabha cross section, and it can be measured, in principle, independently. So, as a result we have a sum of positive

quantities, which is important for Monte Carlo simulations. The 13 contributions are given below.

**1. The (quasi-)elastic kinematics.**

Here we take into account the Born cross section with virtual loop corrections and soft photons. The energy of soft photons does not exceed  $\Delta\varepsilon$ . The parameter  $\Delta \ll 1$  is auxiliary, the final result (the total sum) should not depend on its value. That provides an additional check of our calculation.

$$\frac{d\sigma_1}{d\varphi dc} = \frac{d\tilde{\sigma}(1, 1)}{d\varphi dc} \left\{ 1 + \frac{\alpha}{\pi} K_{SV} + 4\mathcal{D}_\Delta^{[\alpha]} + 6(\mathcal{D}_\Delta^{[\alpha]})^2 + 4\mathcal{D}_\Delta^{[\alpha^2]} - \frac{8\alpha}{\pi} \ln \Delta \ln(\cot(\frac{\theta}{2})) \right\}. \quad (6)$$

**2.  $\mathbf{k} \parallel \mathbf{p}_1$ : one (two) photons along the initial electron momentum.**

In this case we observe emission of a hard collinear photon inside a narrow cone along the direction of motion of the initial electron. The auxiliary parameter  $m_e/\varepsilon \ll \theta_0 \ll 1$  defines the cone:  $\widehat{\mathbf{k}\mathbf{p}_1} < \theta_0$ . We suppose that the parameter is less than the angular resolution of the detector. And therefore we are not going distinguish the situations of one or two photon emission in this cone. We just sum up the energies and momenta, if they are two.

$$\frac{d\sigma_2}{d\varphi dc} = \int_{\bar{z}_1}^{1-\Delta} dz_1 \frac{d\tilde{\sigma}(z_1, 1)}{d\varphi dc} \left\{ \mathcal{D}_\Theta^{[\alpha]}(z_1) + C_{\text{ini}}(z_1) + 3\mathcal{D}_\Theta^{[\alpha]}(z_1)\mathcal{D}_\Delta^{[\alpha]} + \mathcal{D}_\Theta^{[\alpha^2]}(z_1) \right\}, \quad (7)$$

The lowest value of  $z_1$  to be defined from the conditions of particle registration. If  $y_{\text{th}}$  is the threshold energy for electron registration, then

$$\bar{z}_1 = \frac{y_{\text{th}}(1+c)}{2-y_{\text{th}}(1-c)}. \quad (8)$$

The compensator for the initial state radiation is

$$C_{\text{ini}}(z) = \frac{\alpha}{2\pi} \left[ \frac{1+z^2}{1-z} \ln \frac{\theta_0^2}{4} + 1 - z \right]. \quad (9)$$

**3.  $\mathbf{k} \parallel \mathbf{p}_2$ : one (two) photons along the initial positron momentum.**

This case is completely analogous to the previous one.

$$\frac{d\sigma_3}{d\varphi dc} = \int_{\bar{z}_2(z_1=1)}^{1-\Delta} dz_2 \frac{d\tilde{\sigma}(1, z_2)}{d\varphi dc} \left\{ \mathcal{D}_\Theta^{[\alpha]}(z_2) + C_{\text{ini}}(z_2) + 3\mathcal{D}_\Theta^{[\alpha]}(z_2)\mathcal{D}_\Delta^{[\alpha]} + \mathcal{D}_\Theta^{[\alpha^2]}(z_2) \right\}. \quad (10)$$

The lowest limit for  $z_2$  is defined by

$$\bar{z}_2 = \frac{z_1 y_{\text{th}}(1-c)}{2z_1 - y_{\text{th}}(1+c)}, \quad (11)$$

where one has to imply  $z_1 = 1$ .

4.  $\mathbf{k} \parallel \mathbf{p}'_1$ : one (two) photons along the final electron momentum.

$$\frac{d\sigma_4}{d\varphi dc} = \frac{d\tilde{\sigma}(1, 1)}{d\varphi dc} \int_{y_{\text{th}}}^{1-\Delta} dy_1 \left\{ \mathcal{D}_{\Theta}^{[\alpha]}(y_1) + C_{\text{fin}}(y_1) + 3\mathcal{D}_{\Theta}^{[\alpha]}(y_1)\mathcal{D}_{\Delta}^{[\alpha]} + \mathcal{D}_{\Theta}^{[\alpha^2]}(y_1) \right\}. \quad (12)$$

The compensator for the final state radiation is

$$C_{\text{fin}}(y) = \frac{\alpha}{2\pi} \left[ \frac{1+y^2}{1-y} \left( \ln \frac{\theta_0^2}{4} + 2 \ln y \right) + 1 - y \right]. \quad (13)$$

5.  $\mathbf{k} \parallel \mathbf{p}'_2$ : one (two) photons along the final positron momentum.

$$\frac{d\sigma_5}{d\varphi dc} = \frac{d\tilde{\sigma}(1, 1)}{d\varphi dc} \int_{y_{\text{th}}}^{1-\Delta} dy_2 \left\{ \mathcal{D}_{\Theta}^{[\alpha]}(y_2) + C_{\text{fin}}(y_2) + 3\mathcal{D}_{\Theta}^{[\alpha]}(y_2)\mathcal{D}_{\Delta}^{[\alpha]} + \mathcal{D}_{\Theta}^{[\alpha^2]}(y_2) \right\}. \quad (14)$$

6.  $\mathbf{k}_a \parallel \mathbf{p}_1, \mathbf{k}_b \parallel \mathbf{p}_2$ : one photon along the initial electron momentum and one photon along the initial positron momentum.

$$\frac{d\sigma_6}{d\varphi dc} = \int_{\bar{z}_1}^{1-\Delta} dz_1 \int_{\bar{z}_2}^{1-\Delta} dz_2 \frac{d\tilde{\sigma}(z_1, z_2)}{d\varphi dc} \mathcal{D}_{\Theta}^{[\alpha]}(z_1) \mathcal{D}_{\Theta}^{[\alpha]}(z_2). \quad (15)$$

7.  $\mathbf{k}_a \parallel \mathbf{p}'_1, \mathbf{k}_b \parallel \mathbf{p}'_2$ : one photon along the final electron momentum and one photon along the final positron momentum.

$$\frac{d\sigma_7}{d\varphi dc} = \frac{d\tilde{\sigma}(1, 1)}{d\varphi dc} \int_{y_{\text{th}}}^{1-\Delta} dy_1 \int_{y_{\text{th}}}^{1-\Delta} dy_2 \mathcal{D}_{\Theta}^{[\alpha]}(y_1) \mathcal{D}_{\Theta}^{[\alpha]}(y_2). \quad (16)$$

8.  $\mathbf{k}_a \parallel \mathbf{p}_1, \mathbf{k}_b \parallel \mathbf{p}'_1$ : one photon along the initial electron momentum and one photon along the final electron momentum.

$$\begin{aligned} \frac{d\sigma_8}{d\varphi dc} &= \int_{\bar{z}_1}^{1-\Delta} dz_1 \frac{d\tilde{\sigma}(z_1, 1)}{d\varphi dc} \mathcal{D}_{\Theta}^{[\alpha]}(z_1) \int_{y_{\text{th}}/Y_1}^{1-\Delta} dy_1 \mathcal{D}_{\Theta}^{[\alpha]}(y_1), \\ Y_1 &= \frac{2z_1}{z_1 + 1 - c(z_1 - 1)}. \end{aligned} \quad (17)$$

9.  $\mathbf{k}_a \parallel \mathbf{p}_1, \mathbf{k}_b \parallel \mathbf{p}'_2$ : one photon along the initial electron momentum and one photon along the final positron momentum.

$$\begin{aligned} \frac{d\sigma_9}{d\varphi dc} &= \int_{\bar{z}_1}^{1-\Delta} dz_1 \frac{d\tilde{\sigma}(z_1, 1)}{d\varphi dc} \mathcal{D}_{\Theta}^{[\alpha]}(z_1) \int_{y_{\text{th}}/Y_2}^{1-\Delta} dy_2 \mathcal{D}_{\Theta}^{[\alpha]}(y_2), \\ Y_2 &= \frac{z_1^2 + 1 - c(z_1^2 - 1)}{z_1 + 1 - c(z_1 - 1)}. \end{aligned} \quad (18)$$

10.  $\mathbf{k}_a \parallel \mathbf{p}_2, \mathbf{k}_b \parallel \mathbf{p}'_1$ : one photon along the initial positron momentum and one photon along the final electron momentum.

$$\begin{aligned} \frac{d\sigma_{10}}{d\varphi dc} &= \int_{\bar{z}_2}^{1-\Delta} dz_2 \frac{d\tilde{\sigma}(1, z_2)}{d\varphi dc} \mathcal{D}_{\Theta}^{[\alpha]}(z_2) \int_{y_{\text{th}}/Y_1}^{1-\Delta} dy_1 \mathcal{D}_{\Theta}^{[\alpha]}(y_1), \\ Y_1 &= \frac{2z_2}{z_2 + 1 - c(1 - z_2)}. \end{aligned} \quad (19)$$

11.  $\mathbf{k}_a \parallel \mathbf{p}_2, \mathbf{k}_b \parallel \mathbf{p}'_2$ : one photon along the initial positron momentum and one photon along the final positron momentum.

$$\begin{aligned} \frac{d\sigma_{11}}{d\varphi dc} &= \int_{\bar{z}_2}^{1-\Delta} dz_2 \frac{d\tilde{\sigma}(1, z_2)}{d\varphi dc} \mathcal{D}_{\Theta}^{[\alpha]}(z_2) \int_{y_{\text{th}}/Y_2}^{1-\Delta} dy_2 \mathcal{D}_{\Theta}^{[\alpha]}(y_2), \\ Y_2 &= \frac{z_2^2 + 1 - c(1 - z_2^2)}{z_2 + 1 - c(1 - z_2)}. \end{aligned} \quad (20)$$

12.  $\mathbf{p}'_1 \parallel \mathbf{p}'_2$ : both the final electron and positron go together back-to-back to a hard photon.

This contribution works only if no any cut-off on acollinearity is imposed. Its kinematics is just the one of the process of annihilation into two photons (one of which is converted then into an electron-positron pair with low invariant mass). It gives a large logarithm, when the angle between the pair components is small.

$$d\sigma_{12} = d\varphi dc \int_{y_{\text{th}}}^{1-y_{\text{th}}} dz \frac{\alpha^3}{2\pi s} \frac{1+c^2}{1-c^2} \left( \ln \Delta_1 + L - 2 \ln 2 - \frac{5}{3} \right) (z^2 + (1-z)^2), \quad (21)$$

where  $z$  denotes the energy fraction of the final electron, the new auxiliary parameter  $\Delta_1$  bounds the energy of the hard photon for this contribution:  $1 > \omega > \varepsilon(1 - \Delta_1)$ . The opposite condition is to be implied in the last ( $WT$ ) contribution in order to cancel out  $\Delta_1$ .

### 13. One hard photon at large angle in respect to all other particle momenta

$$\frac{d\sigma_{13}}{d\varphi dc} = \frac{\alpha^3}{2\pi^2 s} \int_{\Delta}^{1-\Delta_1} dx \int_{-1}^1 dc_2 \int_0^{2\pi} d\varphi_2 \frac{WT}{4} \Gamma' \Theta. \quad (22)$$

Letter  $\Theta$  denotes the general restrictions on the phase space: polar angles of the photon in respect to any other momentum should be more than  $\theta_0$ ; it should also include the cut-off on acollinearity for the final charged particles, if it is imposed.

$$\begin{aligned} W &= \frac{s}{\chi_+ \chi_-} + \frac{s_1}{\chi'_+ \chi'_-} - \frac{t}{\chi_- \chi'_-} - \frac{t_1}{\chi_+ \chi'_+} + \frac{u}{\chi_- \chi'_+} + \frac{u_1}{\chi_+ \chi'_-}, \\ T &= \frac{ss_1(s^2 + s_1^2) + tt_1(t^2 + t_1^2) + uu_1(u^2 + u_1^2)}{ss_1 tt_1}, \\ s &= 4, \quad t = -2Y_1(1 - c), \quad u = -2Y_2(1 + c_3), \\ s_1 &= 2Y_1(2 - x + xc_1) = 4(1 - x), \quad t_1 = -2Y_2(1 - c_3), \quad u_1 = -2Y_1(1 + c), \\ \chi_- &= x(1 - c_2), \quad \chi_+ = x(1 + c_2), \quad \chi'_- = xY_1(1 - c_1), \quad \chi'_+ = xY_2(1 - c_5), \\ Y_1 &= \frac{2(1 - x)}{2 - x(1 - c_1)}, \quad Y_2 = 2 - Y_1 - x, \\ \Gamma' &= \frac{d\Gamma}{\varepsilon^2 d\varphi dc d\varphi_2 dc_2 dx} = \frac{xY_1}{2 - x + xc_1}. \end{aligned} \quad (23)$$

The current version of the program contains the option to take into account the vacuum polarization affect in this contribution [7].

The complete final state kinematics can be defined in each contribution according to the general formulae given in Ref. [7].

## 3 Event generator structure

An original algorithm for event generation was applied in the code. The steps are as follows.

**Step 1.** At first we perform the numerical integration of the 13 contributions over the phase space, where only the most general cuts are applied:

$$\sigma_i = \int \frac{d\sigma_i}{d\Gamma_i} d\Gamma_i, \quad i = 1, \dots, 13. \quad (24)$$

In this way we obtain the relative weights of the 13 contributions and at the same moment the absolute value of the total cross section:

$$U_i = \frac{\sigma_i}{\sigma_{\text{tot}}}, \quad \sigma_{\text{tot}} = \sum_{i=1}^{13} \sigma_i. \quad (25)$$

**Step 2.** The total ordered number of events to be generated  $N_{\text{tot}}$  is shared between the 13 contributions according their relative contributions to the total cross section. That is done as follows. The main subroutine of the generator is called  $N_{\text{tot}}$  times. Each time we choose one of the 13 kinematical regions according to the value of a random number  $r$ , by comparing it with the relative weights  $w_i$ :

$$\begin{aligned} \text{if } \sum_{k=1}^j U_k < r \leq \sum_{k=1}^{j+1} U_k, \quad & \text{then } i = j; \\ \text{if } r \leq U_1, \quad & \text{then } i = 1. \end{aligned} \quad (26)$$

**Step 3.** Now a set of kinematical variables  $v_n$  for an event of the chosen contribution is generated. For each particular differential distribution  $d\sigma_i/d\Gamma_i$  we use a specific change of variables in order to make the distribution more flat. The weight of the event is defined by the formula:

$$w_n = \frac{d\sigma_i(v_n)}{d\Gamma_i} \frac{N_{\text{tot}}}{\sigma_{\text{tot}}}. \quad (27)$$

Calculating the value of  $d\sigma_i$  we apply the same cuts and conditions as while the numerical integration.

**Step 4.** Because the differential distributions are rather complicated, we were not able to find a substitution to make them completely flat. So, the weights, obtained in the third step, can be different from unit. Here we do the following trick: generate a random number  $r$  and define the number of corresponding unweighted events as the integer part of  $w_n + 1 - r$ . If the obtained number  $m_n$  is more than 1, we use the rotation symmetry and distribute the events uniformly in the polar angle of the scattered electron.

**Step 5.** At this step we can analyse the events generated in the previous step and apply additional cuts, if required. We can also record the events for future processing.

**Step 6.** After we executed the steps 2–4  $N_{\text{tot}}$  times, we can compare the results of numerical and Monte Carlo procedures. That provides a control of the technical precision of the code. Namely, we compare the total number of unweighted events with the ordered one:

$$\sum_{n=1}^{N_{\text{tot}}} m_n \approx N_{\text{tot}}. \quad (28)$$

At the same moment this means that the value of the cross section for the generated events is close to the one obtained by the numerical integration. Such a comparison is also done for each of 13 contributions separately. The technical precision can be improved by increasing of the total number of events and also by tuning parameters of the program for a concrete task.

## 4 Flags and parameters

The reading of flags and parameters is performed by means of the standard `FFREAD` subroutine, which is called from the `PACKLIB` [10].



The code contains several flags which can switch between different options in the physical base and in the generation procedure. Below we describe the most important flags.

#### ICOR

ICOR=0: calculations only at the Born level  
 ICOR=1: Born + LLA corrections  
 ICOR=2: Born + LLA corrections +  $K$ -factor  
 ICOR=3: Born + LLA corrections +  $K$ -factor + large-angle photon

#### IORD

IORD=0: calculations only at the Born level  
 IORD=1:  $\mathcal{O}(\alpha)$  RC are taken into account  
 IORD=2: higher order LLA corrections are included

#### IVPOL

IVPOL=0:  $\alpha_{QED}$  is not running  
 IVPOL=1: vacuum polarization by leptons and hadrons is accounted

The  $\Phi$ -meson contribution to the photon virtual propagator is realized in the program as a part of the vacuum polarization function. This option can be switched on/off by the flag IPHI=1/0.

The main parameters to be set by user are:

EB: the beam energy in GeV;  
 TETN: the minimal electron scattering angle in radian;  
 TETX: the maximal electron scattering angle in radian;  
 TEPN: the minimal positron scattering angle in radian;  
 TEPX: the maximal positron scattering angle in radian;  
 NEVE: the number of events to be generated;  
 TACO: the minimal allowed angle between the outgoing electron and positron in radian;

In order to take into account vacuum polarization in the 13th contribution one has to set IEWT=1 and IVWT=1. This corrections is a part of the second order next-to-leading contributions ( $\sim \mathcal{O}(\alpha^2 L)$ ), but it might be really important in the region close to resonance peaks.

## 5 Conclusions

The presented formulae are valid for the electron–positron colliders of moderately high energies below 3 GeV. In order to expand them for higher energies we take into account  $Z$ -boson exchange and the relevant electroweak radiative corrections [9], but the corresponding option of the code will be described elsewhere.

In Table 1 we give the results for a rather simple configuration: beam energy 0.5 GeV, scattering angles for the both positron and electron lie between  $15^\circ$  and  $165^\circ$ , minimal angle between the final particles  $30^\circ$ , threshold for electron registration 50 MeV. For the Table we

Table 1: Results of numerical calculations

	$\sigma_{\text{Born}}$	$\sigma_{\text{LLA}}^{(1)}$	$\sigma_{\text{LLA+K}}^{(1)}$	$\sigma^{(1)}$	$\sigma^{(2)}$
	without vacuum polarization				
Numer. int., mbarn	13.501	13.360	13.023	13.325	13.303
Monte Carlo, mbarn	13.501	13.359	13.024	13.324	13.298
$\delta_{\text{RC}}, \%$	—	−1.05	−3.55	−1.31	−1.47
	with vacuum polarization				
Numer. int., mbarn	13.749	13.604	13.261	13.631	13.638
Monte Carlo, mbarn	13.749	13.603	13.259	13.627	13.632
$\delta_{\text{RC}}, \%$	1.83	0.76	−1.78	0.96	1.01

generated  $10^8$  unweighted events. The last line of the Table represents the relative difference of the cross section in the corresponding approximation in respect to the Born one:

$$\delta_{\text{RC}} = \frac{\sigma_i - \sigma_{\text{Born}}}{\sigma_{\text{Born}}} 100\%. \quad (29)$$

Further, in the Table we denote:  $\sigma^{\text{Born}}$  is the pure Born level QED cross section;  $\sigma_{\text{LLA}}^{(1)}$  is the cross section with the  $\mathcal{O}(\alpha)$  LLA corrections; in  $\sigma_{\text{LLA+K}}^{(1)}$  the  $\mathcal{O}(\alpha)$   $K$ -factor due to soft and virtual photons is included; quantity  $\sigma^{(1)}$  includes the complete  $\mathcal{O}(\alpha)$  set of corrections; the complete set of the first order corrections plus the LLA second order effects  $\sigma^{(2)}$  are presented in the last column.

The resulting precision of the code for the description of large-angle Bhabha scattering in the typical conditions of electron–positron colliders of energy about a few GeVs is estimated to be 0.2% [7]. The uncertainty can be decreased. In particular, an extended program for calculations of second order next-to-leading RC to large-angle Bhabha scattering is in progress [11]. From the other hand, the uncertainty should be re-estimated taking into account the concrete experimental conditions and comparisons with other codes.

Comparisons with other available Monte Carlo event generators for Bhabha scattering are in progress. The next step of the code development will be the explicit generation of radiated pairs. The third order leading logarithmic corrections will be also taken into account. The current version of the program does not include the strange effect of a deviation from the leading logarithmic approximation in the  $\mathcal{O}(\alpha^2 L^2)$  order, which was recently found in Ref. [12].

The presented version of the code is dealing with the large-angle Bhabha scattering. That does not mean that the code can not be used for small-angles, but that it does not include the complete set of the second-order next-to-leading corrections, which are known only for the small-angle limiting case [13]. An extended version of the code, which will provide also the small-angle Bhabha event generation, to be described elsewhere.

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